
**ISIJ 2020, Math Tournament,
July 8, 2020, Yandex Contest**

1. CIRCLES

Answer: **62**

Let S_1 and S_2 be the areas of the larger and smaller circles, and the area of the common part of the circles is equal to S . Then the areas of the parts specified in the condition are equal to $S_1 - S$ and $S_2 - S$. Therefore, the desired difference is equal to $(S_1 - S) - (S_2 - S) = S_1 - S_2 = \pi * (6^2 - 4^2) = \pi * 20 = 62,8\dots$

2. CAKE

Answer: **3 3**

This is the center of the parallelogram, i.e. intersection point of diagonals. Rabbit can't get more than half of the cake. Any straight line passing through the center of a parallelogram divides it into two equal parts.

3. TWO POLYHEDRONS

Answer: **30**

By Euler's formula: vertices + facets - edges = 4; $38 + f - 64 = 4$; $f = 30$.

If anybody does not know this formula: one of heuristical ways:

- if such a task is given then it has a solution;
- if both polyhedra are equal then
- vertices = 19; edges = 32 for each.

A regular polygon with 19 vertices has 19 edges and 2 facets (including the outer one). Adding 13 (non-intersecting) chords we obtain 32 edges and 15 facets.

4. MAX PRODUCT

Answer: **162**

After several experiments we find: $3*3*3*3*2=162$.

5. SUM OF ODD NUMBERS

Answer: **44**

The sum of the first n odd numbers is equal to n^2 . For the boundary of the 2020 range, the sum of odd numbers that do not exceed the specified boundary must be no more than an integer part of the square root of 2020. In other words, there are 44 such numbers in the range $[1; 2020]$, and their sums correspondingly have values: $1^2, 2^2, \dots, 44^2$.

6. MAX ROOT

Answer: **100**

Calculate the power of the variable x in the left part:

$$S = \frac{2}{3} + \frac{1}{3} \cdot S, \text{ so } S = 1.$$

(You can use the formula for the sum of the terms of an infinitely decreasing geometric progression.) Hence $x^1 = 100$.

7. CHESS

Answer: **G6 C6**

Unique move Rg6-c6!

8. SOFA AND WARDROBE

Answer: **14**

Sequence of moves: T-T-S-T-W-T-S-T-T-W-T-S-T-T.

9. SUM OF PERMUTATIONS

Answer: **3 999 960**

There are only $5! = 120$ ways to rearrange numbers in the number 12345. Among these methods, exactly in the fifth part (i.e. in 24 cases), the number 1 is in the first place. The same is true for any number and for any place. Therefore, the required sum is equal to:

$$\begin{aligned} &24(10000 + 1000 + 100 + 10 + 1 + 20000 + 2000 + 200 + 20 + 2 + 30000 + 3000 + 300 + \\ &30 + 3 + 40000 + 4000 + 400 + 40 + 4 + 50000 + 5000 + 500 + 50 + 5) = \\ &24(11111 + 22222 + 33333 + 44444 + 55555) = 24 \cdot 11111 \cdot (1 + 2 + 3 + 4 + \\ &5) = 3\,999\,960. \end{aligned}$$

10. NUMBER OF FRACTIONS

Answer: **256**

In any fraction, x_1 will always be in the numerator, and x_2 be in the denominator. Each of the 8 letters x_3, x_4, \dots, x_{10} can appear independently of the others in the numerator or denominator. Therefore, you will get a total of 2^{10-2} fractions.